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Dynamical Generation of Mass

Vicente Delgado
Departamento de Física Fundamental y Experimental,
Universidad de La Laguna, E-38203 La Laguna, Tenerife, Spain.
E-mail: vdelgado@ull.es

Abstract

An alternative for the Higgs mechanism is proposed. It predicts the appearance in the broken phase of a scalar background field which may be interpreted as describing an almost uniform (i.e., homogeneous and isotropic) superfluid condensate of decoupled Higgs bosons. Quantum fields acquire mass as a consequence of nonperturbative interactions with those particles condensed in the zero-momentum state (which constitutes the physical vacuum of the theory) giving rise in turn to the appearance of density fluctuations. This mechanism has therefore remarkable cosmological implications.

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Spontaneous symmetry breaking is one of the most widely observed phenomena in nature [1]. It occurs when the ground state of a system corresponding to a particular solution of the equations of motion exhibits a lower symmetry than the Lagrangian density.

At low temperatures, when thermal fluctuations become irrelevant, most physical systems undergo phase transitions toward stable configurations which do not exhibit the full underlying symmetry of the equations of motion that govern their dynamical evolution. Quantum fluctuations are therefore expected to play an important role in the origin of the instability of the symmetric phase. In particular it is well known that the macroscopic occupation that takes place in quantum condensation phenomena may be interpreted as a process of symmetry breaking [2–5]. The acumulation of particles in the ground state induces a non-vanishing vacuum expectation value (VEV) for the corresponding scalar field, which then acts as the order parameter required for a complete characterization of the broken phase.

From the perspective of Theoretical Physics spontaneous symmetry breaking turns out to be of particular interest because it can provide, in principle, a conceptually simple mechanism for explaining the complexity of nature starting from a highly symmetric initial state. Not surprinsingly, this phenomenon is one of the basic underlying ideas of both Unified Gauge Theories and Inflationary Cosmology. In fact, the only renormalizable gauge theories with massive vector bosons are gauge theories with spontaneous symmetry breaking [6].

In modern Unified Theories [7] massive intermediate bosons are properly introduced by breaking the symmetry by means of the Higgs mechanism [8,9]. It basically consists in the introduction of a complex scalar field $\phi(x)$ subject to an effective potential of the form

$$V(\phi) = \mu^2 \phi^+ \phi + \lambda (\phi^+ \phi)^2 \tag{1}$$

with $\mu^2 < 0$. The necessity of introducing a negative mass squared is the price one has to pay in order to generate a nonzero stable configuration $\langle \phi \rangle_0$.

In the present paper it is proposed an alternative for the Higgs mechanism where spontaneous symmetry breaking has its origin in the dynamics of the scalar field ϕ in the presence of gauge (and matter) fields. The symmetry breaks once a particular solution of the equations of motion is chosen among a (one-parameter) family of possible solutions.

Even though it turns out to be more natural in the presence of fermion fields [10], it is convenient to consider an abelian U(1) gauge theory in order to illustrate the mechanism in its simplest context. This theory, apart from reflecting more clearly the physics involved, has the additional interest that in this case the present mechanism basically represents a covariant generalization of the London theory of superconductivity. And this analogy with superconductivity proves to be very useful in order to gain valuable insights about the nature of the process of symmetry breaking. Furthermore, it turns out that the low-energy effective Lagragian density governing the decoupled scalar sector takes the same form as in the case of a SU(3)×SU(2)×U(1) gauge theory [11], so that most of the cosmological consequences (directly related to that sector) may already be extracted by studying this simple case. For these reasons we shall focus in what follows on a U(1) gauge theory. Specific application to the Standard Model will be considered elsewhere [11,12].

The U(1) gauge-invariant Lagrangian density for a complex scalar field $\phi(x)$ reads [13]

$$\mathcal{L} = (D_\mu \phi)^+ (D^\mu \phi) - \mu^2 \phi^+ \phi - \frac{1}{4} \mathrm{F}_{\mu\nu} \mathrm{F}^{\mu\nu} \tag{2}$$

where now $\mu^2 > 0$ is the mass of the scalar field, $\mathrm{F}_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$ is the U(1) field strength tensor, and the gauge-covariant derivative is given by $D_\mu = \partial_\mu - igB_\mu$.

It should be emphasized that even though it is possible to break the symmetry starting with a Lagrangian density containing a quartic scalar self-interaction, $\lambda(\phi^+\phi)^2$, we explicitly take $\lambda \equiv 0$. The motivation for such a choice lies mainly in the fact that taking the limit $\lambda \to 0$ clearly leads to physical transparency. Since a ϕ^4 term is necessary in order to guarantee perturbative renormalizability in the scalar sector of the theory, this implies that we choose to work with a nonrenormalizable scalar sector. Renormalizability has to be demanded only in the case of a fundamental theory which must remain valid at arbitrary high energies, and there exists no indication that the scalar sector of the Standard Model must satisfy such a requirement. In fact, superconductivity provides indications just in the opposite direction. We shall therefore assume that at low temperatures the scalar field $\phi(x)$ provides an adequate description of a more fundamental physical structure, in such a way that the Lagrangian density (2) remains valid up to temperatures of order T_c. Since we are interested in physics in the broken-symmetry phase, at $T \simeq 0$, in this paper we shall not concern ourselves about the fundamental nature of μ excitations. In fact, as we shall see, according to the present mechanism the process of mass generation turns out to be a low-temperature collective phenomenon where mass appears as a physical parameter reflecting the cumulative effect of nonperturbative interactions with those scalar particles macroscopically condensed in the zero-momentum state. Thus, one expects the structure of scalar excitations to be irrelevant for the process of generation of mass. Our physical problem therefore will reduce to give mass to the vector bosons preserving (hidden) gauge invariance.

In order to exploit the symmetry of the problem it turns out to be convenient parametrizing $\phi(x)$ in polar form

$$\phi(x) = \frac{1}{\sqrt{2}}\rho(x) \exp i\xi(x) \tag{3}$$

where $\rho(x)$ and $\xi(x)$ are real scalar fields. In the unitary gauge, which clearly reflects the particle content of the theory, we have $\phi(x) \to \rho(x)/\sqrt{2}$, and the Lagrangian density can be written

$$\mathcal{L} = \frac{1}{2}(\partial_\mu\rho)(\partial^\mu\rho) + \frac{1}{2}g^2 A_\mu A^\mu \rho^2 - \frac{1}{2}\mu^2\rho^2 - \frac{1}{4}\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu} \tag{4}$$

with $A_\mu(x) = B_\mu(x) - (1/g)\partial_\mu\xi(x)$.

The Euler-Lagrange equations of motion governing the coupled dynamics of the fields, are

$$\partial_\mu\partial^\mu\rho(x) = -\mu^2\rho(x) + g^2A^2(x)\rho(x) \tag{5}$$
$$\partial_\alpha\partial^\alpha A^\nu - \partial^\nu(\partial_\beta A^\beta) = -g^2A^\nu\rho^2 \tag{6}$$

where the source of the gauge field, $\rho^2(x)A^\mu(x)$, is a conserved current

$$\partial_\mu(\rho^2 A^\mu) = 0 \tag{7}$$

In what follows we will show that the equations of motion (5)–(6) admit a family of solutions which verify

$$A^{\mu}(x) = \frac{J^{\mu}(x)}{\rho^2(x)} \tag{8}$$

where $J^{\mu}(x)$ is a conserved current independent of $\rho(x)$, and a subset of these solutions leads to a theory with broken symmetry and massive gauge bosons.

Note that Eq.(8) is nothing but a covariant generalization of the well-known London formula [14], relating the vector potential with the supercurrents. The London formula, which can be obtained from the BCS theory in the long wavelength limit [15], plays a fundamental role in superconductivity, and is essentially a consequence of the *rigidity* of the electronic superfluid with respect to perturbations [16]. It has the importance that it leads to the Meissner effect, which, as first noted by Anderson [17], basically reflects the fact that because of nonperturbative interactions with the medium the gauge quanta dynamically acquire mass in a superconductor.

As we shall see, the solutions (8) also lead, in particular, to a rigid (i.e., unperturbed) scalar field $\rho(x)$, which in the present context must be interpreted as a decoupled background field. In fact, the contribution of $A^{\mu}(x)$ to the dynamical evolution of $\rho(x)$ becomes suppressed by inverse powers of $\langle\rho\rangle_0$, so that, in the weak coupling phase, corresponding to $\langle\rho\rangle_0 \to \infty$, the scalar field becomes virtually unaffected by the presence of the gauge field, and plays the role of a background field. Therefore, the solutions we are interested in actually describe a nonperturbative broken phase with massive gauge quanta in a background scalar field. This fact, in turn, helps us to understand the physics involved in the process. Indeed, Eq.(8), basically showing that in the nonperturbative phase $A^{\mu}(x)$ develops a dependence of the form $1/\rho^2$, can be better understood if one takes into account that eventually $\rho(x)$ behaves as a classical background field (consider in particular a cosmological scenario).

By substituting Eq.(8) into Eq.(5) we obtain

$$\partial_{\alpha}\partial^{\alpha}\rho(x) = -\frac{\partial}{\partial\rho}V_{\text{eff}}(x) \tag{9}$$

where

$$V_{\text{eff}}(x) = \frac{1}{2}\mu^2\rho^2(x) + g^2\frac{J^2(x)}{2\rho^2(x)} \tag{10}$$

As we shall see, the fact that the potential energy density $V_{\text{eff}}(\rho; J^2)$ depends on the conserved current only through the composite scalar operator $J^2(x)$, turns out to be of particular importance in our treatment.

In field theories the vacuum $|0\rangle$ is defined as the ground state of the theory. The property of translational invariance that this state must possess requires the vacuum expectation values of physical quantities to be constants independent of space-time coordinates. Therefore, from Eq.(9) one finds that the vacuum $|0\rangle$ must verify

$$\langle 0|\frac{\partial}{\partial\rho}V_{\text{eff}}|0\rangle = 0 \tag{11}$$

in agreement with physical intuition. Making use of the independence of $J^{\mu}(x)$ on $\rho(x)$, this condition reads

$$\mu^2\langle\rho\rangle_0 - g^2\frac{\langle J^2\rangle_0}{\langle\rho^3\rangle_0} = 0 \tag{12}$$

Note that gauge (and matter [11]) fields make a nonperturbative contribution in the selection of the ground state $|0\rangle$. Eq.(12) also shows that the properties of the vacuum depend on quantum fluctuations of physical currents, the quantum nature of the system playing therefore an essential role in the process.

Different values of $\langle J^2\rangle_0$ characterize different possible solutions and hence different physical vacua. And due to the fact that $\langle J^2\rangle_0$ is a constant it is not possible to smoothly pass from one to another solution, so that we have a one-parameter family of stable solutions, and the U(1) symmetry breaks once a particular one is taken.

The subset of solutions we are interested in are those with $\langle J^2\rangle_0 \equiv J_0^2 > 0$, because as can be seen from Eq.(12), in the presence of vacuum fluctuations of physical currents, the nonlinear terms in the equations of motion induce a nonzero stable configuration $\langle\rho\rangle_0 \equiv v \neq 0$. In this case we have

$$\mu^2\langle\rho\rangle_0\langle\rho^3\rangle_0 = g^2\langle J^2\rangle_0 \tag{13}$$

On the other hand, Eq.(8) leads to

$$\langle J^2\rangle_0 = \langle\rho^4\rangle_0\langle A_\mu A^\mu\rangle_0 \tag{14}$$

so that, the solutions with $\langle\rho\rangle_0 \neq 0$ satisfy

$$\mu^2\langle\rho\rangle_0\langle\rho^3\rangle_0 = g^2\langle\rho^4\rangle_0\langle A_\mu A^\mu\rangle_0 \tag{15}$$

Notice that a similar equation, relating the mass of the scalar field with the vacuum fluctuations of the gauge field, can be directly obtained from the initial equations of motion. Indeed, taking the VEV of Eq.(5) one finds

$$\mu^2\langle\rho\rangle_0 = g^2\langle A_\mu A^\mu \rho\rangle_0 \tag{16}$$

However, physical considerations lead us to look for solutions satisfying in addition

$$\langle A_\mu A^\mu \rho\rangle_0 \simeq \langle A_\mu A^\mu\rangle_0\langle\rho\rangle_0 \tag{17}$$

This condition, which obviously holds in a perturbative phase, is necessary in order to allow us the construction of a Hilbert-space basis with a simple interpretation in terms of quanta of the $A^\mu(x)$ and $\rho(x)$ fields, which eventually must be considered as the physically relevant degrees of freedom. Making use of this requirement in Eq.(16), we find that there exists physically meaningful solutions with $\langle\rho\rangle_0 \neq 0$ only if

$$\langle A_\nu A^\nu\rangle_0 \simeq \mu^2/g^2 \tag{18}$$

This equation basically states that for the process to take place, μ particles should be created from energy fluctuations of gauge fields in the ground state, and to the extent that μ^2 is an externally given parameter it may represent a strong constraint.

Note on the other hand that the existence of solutions with a simple interpretation in terms of quanta of $A^\mu(x)$ and $\rho(x)$ is not, in general, compatible with Eq.(8). In fact, the

existence of such solutions is again a consequence of the rigidity of the scalar field which, as will be seen, allows us to absorb the nonperturbative part of the interaction into a redefinition of the fields, leading to an effective theory with a massive gauge field interacting with those scalar particles situated above the ground state. Comparing Eqs.(15) and (18) we find that consistency demands

$$\langle\rho^4\rangle_0 \simeq \langle\rho\rangle_0\langle\rho^3\rangle_0 \tag{19}$$

so that, we are led to look for solutions satisfying

$$\langle\rho^n\rangle_0 \simeq \langle\rho\rangle_0^n = v^n \tag{20}$$

which, as can be easily verified, represents a sufficient condition in order for Eq.(17) to hold in the nonperturbative phase described by the solutions (8). This factorization property, which in fact is also implicit in the Higgs mechanism, is characteristic of a system with a macroscopic occupation of the ground state, and in particular implies that the behaviour of the scalar field in the vacuum must be essentially classical in character. Indeed, from Eq.(20) we have

$$\sqrt{\langle\rho^2\rangle_0 - \langle\rho\rangle_0^2} \simeq 0 \tag{21}$$

showing that our solutions are incompatible with the existence of relevant scalar fluctuations regardless of their thermal or quantum nature. This fact justifies a zero-temperature treatment in determining the asymmetric vacuum.

Making use of Eq.(20) we can rewrite Eq.(13) in the form

$$v^4 = \frac{g^2}{\mu^2}\langle J^2\rangle_0 \tag{22}$$

which reflects that the appearance of a nonvanishing VEV $\langle\rho\rangle_0$ is a direct consequence of the existence of vacuum fluctuations of the physical current $J^\mu(x)$.

Notice, on the other hand, that (20) also implies that the vacuum constitutes a stable configuration of $V_{\text{eff}}(\rho; J^2)$. More precisely, from Eqs.(10), (11) and (20) one finds that the VEVs of $\rho(x)$ and $J^2(x)$ minimize the potential energy density

$$\left.\frac{\partial}{\partial\rho}V_{\text{eff}}\right|_{\langle\rangle} \simeq 0 \tag{23}$$

where the notation $\langle\rangle$ stands for $(\rho, J^2) = (v, J_0^2)$. In fact, as can be readily verified, Eq.(23) is nothing but a particular case of the more general relation

$$\left.\frac{\partial^n}{\partial\rho^n}V_{\text{eff}}\right|_{\langle\rangle} \simeq \langle 0|\frac{\partial^n}{\partial\rho^n}V_{\text{eff}}|0\rangle \tag{24}$$

which holds for $n \geq 0$.

As expected, expansion about the stable configuration contributes to simplify considerably the theory. By expanding $V_{\text{eff}}(\rho; J^2)$ about the vacuum

$$
\begin{aligned}
V_{\text{eff}}(x) = V_{\text{eff}}|_{\langle\rangle} &+ \left.\frac{\partial V_{\text{eff}}}{\partial J^2}\right|_{\langle\rangle}(J^2(x)-J_0^2) + \frac{1}{2!}\left.\frac{\partial^2 V_{\text{eff}}}{\partial \rho^2}\right|_{\langle\rangle}(\rho(x)-v)^2 \\
&+ \left.\frac{\partial^2 V_{\text{eff}}}{\partial \rho\, \partial J^2}\right|_{\langle\rangle}(\rho(x)-v)(J^2(x)-J_0^2) + \frac{1}{3!}\left.\frac{\partial^3 V_{\text{eff}}}{\partial \rho^3}\right|_{\langle\rangle}(\rho(x)-v)^3 + \ldots
\end{aligned} \tag{25}
$$

one obtains

$$
\begin{aligned}
V_{\text{eff}}(x) = \mu^2 v^2 &+ 2\mu^2(\rho(x)-v)^2 - \frac{2\mu^2}{v}(\rho(x)-v)^3 + \frac{5\mu^2}{2v^2}(\rho(x)-v)^4 \\
&+ \frac{g^2}{2v^2}(J^2(x)-J_0^2) - \frac{3\mu^2}{v^3}(\rho(x)-v)^5 - \frac{g^2}{v^3}(\rho(x)-v)(J^2(x)-J_0^2) + O(\frac{1}{v^4})
\end{aligned} \tag{26}
$$

where use has been made of Eq.(22). Higher terms are increasingly suppressed by inverse powers of the VEV of the scalar field, so that the physically relevant limit $\langle\rho\rangle_0 \to \infty$ corresponds to a nonperturbative weak coupling phase.

Notice that up to corrections of order $1/v^2$ the effect of gauge interactions is completely absorbed into a redefinition of the low-energy effective scalar theory. Indeed, substitution of Eq.(26) into Eq.(9) leads to

$$
\partial_\alpha \partial^\alpha \rho(x) + m^2(\rho(x)-v) = \lambda(\rho(x)-v)^2 + \ldots \tag{27}
$$

where

$$
m^2 = \left.\frac{\partial^2}{\partial \rho^2} V_{\text{eff}}\right|_{\langle\rangle} = \mu^2 + 3\frac{g^2 l^2}{v^4} = 4\mu^2 \tag{28}
$$

$$
\lambda = -\frac{1}{2}\left.\frac{\partial^3}{\partial \rho^3} V_{\text{eff}}\right|_{\langle\rangle} = 6\frac{g^2 l^2}{v^5} = \frac{6\mu^2}{v} \tag{29}
$$

so that, as previously said, $\rho(x)$ decouples, becoming unaffected by the presence of gauge (and matter [11]) fields. In fact Eq.(27) describes a low-energy mean field theory involving an independent scalar field $\rho(x)$ subject to an effective mean potential which contains the effects of gauge interactions. In particular, according to (24) the mass and coupling constant characterizing the dynamical evolution of $\rho(x)$ are simply given by

$$
m^2 \simeq \langle 0|\frac{\partial^2}{\partial \rho^2} V_{\text{eff}}|0\rangle \tag{30}
$$

$$
\lambda \simeq -\frac{1}{2}\langle 0|\frac{\partial^3}{\partial \rho^3} V_{\text{eff}}|0\rangle \tag{31}
$$

Experience tells us that mean field theories usually describe weakly interacting systems with a long-range order characteristic of macroscopic classical systems. Not surprisingly the low-energy scalar theory defined by Eq.(27) exhibits long-range order too. Indeed, the factorization property (20) implies that the correlation function $\langle 0|\rho(x)\rho(y)|0\rangle$ remains constant over arbitrary large space-time intervals

$$
\langle 0|\rho(x)\rho(y)|0\rangle \simeq v^2 \tag{32}
$$

Such infinite-range behaviour is a characteristic feature of superfluid Bose systems [18] and is a consequence of the macroscopic occupation of a single quantum state. As we shall see below, the scalar field $\rho(x)$ may be interpreted as describing an almost uniform superfluid composed of μ particles mainly condensed in the zero-momentum state, which represents the lowest-energy state and therefore defines the nontrivial vacuum of the theory.

As Eq.(27) shows, in a dynamical description rather than $\rho(x)$ itself the physically relevant quantity is its departure from the vacuum. It is therefore convenient to define a real scalar field with vanishing VEV

$$\eta(x) = \rho(x) - v \tag{33}$$

in terms of which the equations of motion governing the low-energy effective theory finally read

$$(\partial_\alpha \partial^\alpha + m^2)\eta(x) = \lambda \eta^2(x) + \dots \tag{34}$$

$$(\partial_\alpha \partial^\alpha + m_A^2)A^\nu - \partial^\nu(\partial_\beta A^\beta) = j^\nu(x) \tag{35}$$

where

$$j^\nu(x) = -(2gm_A\eta(x) + g^2\eta^2(x))A^\nu(x) \tag{36}$$

Therefore for a particular solution (8) with $J_0^2 > 0$, the scalar field develops a nonvanishing VEV

$$\phi(x) = \frac{1}{\sqrt{2}}(v + \eta(x)) \exp i\xi(x) \tag{37}$$

and the gauge field $A_\mu(x)$ acquires a mass

$$m_A^2 = g^2 v^2 \tag{38}$$

in accordance with the well-known results [13]. Nonzero vacuum fluctuations of $J^\mu(x)$ induce spontaneous breakdown of gauge symmetry, which then becomes hidden in the sense that the particle spectrum of the low-energy effective theory does not display the full symmetry of the original Lagrangian density.

Eq.(34) reflects the fact that the field $\rho(x) = v + \eta(x)$ decouples. Quantum fields exhibit this kind of behaviour in the limiting case where a macroscopically large number of quanta appear in a coherent state, and in such a situation they behave as ordinary classical fields. In particular, $\rho(x)$, which describes μ particles, behaves as a classical scalar field that fluctuates about $\rho(x) = v$ with a mass $m = 2\mu$. Then one expects that a macroscopic physical meaning should be possible to be given to $\rho(x)$. Indeed, it may be interpreted as describing a superfluid Bose condensate of μ particles with vanishing momenta and a particle number density

$$n(x) = \frac{1}{2} m \rho^2(x) \tag{39}$$

In order to see this, let us consider a real scalar field $\rho(x)$ describing a system of almost non-interacting bosons of mass m in a volume $V \to \infty$. At zero temperature most particles

are condensed in the zero-momentum state, which is the lowest-energy state. Under this conditions, because of the fact that in replacing

$$\frac{1}{V}\sum_{\mathbf{k}} \rightarrow \frac{1}{(2\pi)^3}\int d^3\mathbf{k} \tag{40}$$

one neglects the contribution of the zero-momentum state, a continuous treatment turns out to be inadequate. Therefore we shall use in what follows a discrete formulation, which proves to be particularly convenient in treating highly degenerate Bose systems.

In the case of almost non-interacting particles, $\rho(x)$ may be expanded in a Fourier series of plane waves in the form

$$\rho(x) = \sum_{\mathbf{k}} \frac{1}{\sqrt{V\,2k^0}}[a_{\mathbf{k}}e^{-ikx} + a^+_{\mathbf{k}}e^{ikx}] \tag{41}$$

where $k^0 = (m^2 + \mathbf{k}^2)^{1/2}$, $kx = k^0x^0 - \mathbf{kx}$, and $a^+_{\mathbf{k}}$, $a_{\mathbf{k}}$ are the creation and annihilation operators satisfying the usual conmutation relations

$$[a_{\mathbf{k}}, a^+_{\mathbf{k'}}] = \delta_{\mathbf{kk'}} \tag{42}$$

The state of the system, $|\alpha\rangle$, may be completely characterized by giving the occupation numbers, $N_{\mathbf{k}}$, of the different $\mathbf{k}$-momentum states. In particular, the vacuum $|0\rangle$ is simply defined by the condition $N_{\mathbf{k}} = 0$ for all $\mathbf{k} \neq \mathbf{0}$ (i.e., no particles in excited states)

$$|0\rangle \equiv |N_0, 0, \ldots, 0, \ldots\rangle \tag{43}$$

Since at temperature $T \simeq 0$ most particles are condensed in the zero-momentum state, we have

$$N_0 \gg N_{\mathbf{k}\neq\mathbf{0}} \tag{44}$$

On the other hand, because of the macroscopic size of the system ($V \to \infty$) we expect in addition a macroscopic occupation of the lower excited states

$$N_0, \ldots, N_{\mathbf{k}}, \ldots \gg 1 \tag{45}$$

Under these conditions, as first noted by Bogoliubov [19], the corresponding creation and annihilation operators, $a^+_{\mathbf{k}}$ and $a_{\mathbf{k}}$, behave as c-numbers. Indeed, for a macroscopically occupied $\mathbf{k}$-momentum state the relations

$$a_{\mathbf{k}}\,|N_0, \ldots, N_{\mathbf{k}}, \ldots\rangle = \sqrt{N_{\mathbf{k}}}\;|N_0 \ldots, N_{\mathbf{k}} - 1, \ldots\rangle \tag{46}$$

$$a^+_{\mathbf{k}}\,|N_0, \ldots, N_{\mathbf{k}}, \ldots\rangle = \sqrt{N_{\mathbf{k}} + 1}\;|N_0 \ldots, N_{\mathbf{k}} + 1, \ldots\rangle \tag{47}$$

simply reduce to

$$a_{\mathbf{k}}\,|\alpha\rangle \simeq \sqrt{N_{\mathbf{k}}}\,|\alpha\rangle \tag{48}$$

$$a^+_{\mathbf{k}}\,|\alpha\rangle \simeq \sqrt{N_{\mathbf{k}}}\,|\alpha\rangle \tag{49}$$

where we have made use of the fact that the state of the system

$$|\alpha\rangle \equiv |N_0, \ldots, N_{\mathbf{k}}, \ldots\rangle \tag{50}$$

remains virtually unaffected by the addition or substraction of a particle in the **k**-momentum state (the corresponding correction is only of order $1/N_{\mathbf{k}}$). Therefore, the operators $a_{\mathbf{k}}^{+}$ and $a_{\mathbf{k}}$ behave as c-numbers and may be replaced by $\sqrt{N_{\mathbf{k}}}$. Using this approximation in Eq.(41) we obtain

$$\rho(x) = \sqrt{\frac{2n_0}{m}} \cos(mt) + \sum_{\mathbf{k}\neq\mathbf{0}} \frac{1}{\sqrt{V\,2k^0}} [a_{\mathbf{k}} e^{-ikx} + a_{\mathbf{k}}^{+} e^{ikx}] \tag{51}$$

where n_0 denotes the mean number density of the ground state

$$n_0 \equiv \frac{N_0}{V} \tag{52}$$

Eq.(51) may be written in the form

$$\rho(x) = \sqrt{\frac{2n_0}{m}} [1 + O(m^2 t^2)] + \eta(x) \tag{53}$$

where the classical field $\eta(x)$ is given by

$$\eta(x) = \sum_{\mathbf{k}\neq\mathbf{0}} \frac{1}{\sqrt{V\,2k^0}} [a_{\mathbf{k}} e^{-ikx} + a_{\mathbf{k}}^{+} e^{ikx}] \simeq \sum_{\mathbf{k}\neq\mathbf{0}} \sqrt{\frac{2n_k}{k^0}} \cos(k^0 x^0 - \mathbf{k}\mathbf{x}) = \langle\alpha|\eta(x)|\alpha\rangle \tag{54}$$

n_k being the mean number density of the **k**-momentum state. Incidentally, note that $\langle 0|\eta(x)|0\rangle = 0$.

Comparing (33) with (53) in the limit $mt \ll 1$ [10], one is led to identify

$$v = \sqrt{\frac{2n_0}{m}} \tag{55}$$

which provides a physical interpretation for $\langle\rho\rangle_0$. Indeed, this equation states that the VEV of $\rho(x)$ essentially measures the mean number density of particles condensed in the zero-momentum state (vacuum). Eq.(55) also reflects that the limit $v \to \infty$ physically corresponds to a macroscopically occupied ground state, $n_0 \to \infty$. Therefore the factorization property (20), which holds provided that

$$\langle 0|\eta^n(x)|0\rangle \ll v^n \tag{56}$$

is indeed a consequence of the macroscopic occupation of the vacuum. Making use of this property we may rewrite Eq.(55) in the form

$$n_0 \simeq \langle 0|\frac{1}{2} m \rho^2(x)|0\rangle \tag{57}$$

so that one expects the number density of the Bose system in the state $|\alpha\rangle$ to be given by

$$n(x) \equiv \langle\alpha|\frac{1}{2}m\rho^2(x)|\alpha\rangle \simeq \frac{1}{2}m\rho^2(x) \tag{58}$$

where the last step reflects the fact that, to a good approximation, $\rho(x)$ behaves as a c-number. Taking into account that $\langle\alpha|\eta(x)|\alpha\rangle \sim \sum\sqrt{n_k}$ while $v \sim \sqrt{n_0}$, one has, according to Eq.(44),

$$\langle\alpha|\eta(x)|\alpha\rangle \ll v \tag{59}$$

so that (58) can be expressed in the form

$$n(x) \simeq n_0\left[1 + 2\frac{\eta(x)}{v}\right] \tag{60}$$

with $\eta(x)/v \ll 1$. Then, the scalar background field $\rho(x) = v + \eta(x)$ may be interpreted as describing an almost uniform (i.e., homogeneous and isotropic) superfluid condensate of decoupled μ particles with a number density $n(x) \simeq n_0$, the superfluid behaviour being a consequence of the fact that most particles are condensed in the zero-momentum state and therefore do not contribute to the viscosity (neither to the pressure).

Eq.(60) also leads to

$$\eta(x) \simeq \frac{v}{2n_0}(n(x) - n_0) \tag{61}$$

which shows that $\eta(x)$ can be regarded in turn as describing density fluctuations about the mean value n_0. Incidentally, note that according to Eq.(28) the mass of this field corresponds to the energy necessary to create a pair of μ particles.

With the present interpretation the minimun of the potential energy density may be written as

$$V_{\text{eff}}|_{\langle\rangle} = \mu^2 v^2 = \mu n_0 \tag{62}$$

where n_0 is the mean number density of the ground state. Therefore, $V_{\text{eff}}|_{\langle\rangle} \simeq \langle 0|V_{\text{eff}}|0\rangle$ [see Eq.(24)] is nothing but the internal energy of a system of non-interacting μ particles condensed in the zero-momentum state.

On the other hand, according to Eqs.(38) and (55), the gauge boson acquires a mass

$$m_A^2 = \frac{g^2 n_0}{\mu} \tag{63}$$

which also agrees with what one would expect. Indeed m_A^{-1} coincides with the London penetration depth for a gauge field in a condensate of μ particles with number density n_0. Therefore, the physical parameter m_A^2 accounts for the cumulative effect of nonperturbative interactions with those scalar particles condensed in the zero-momentum state, and the low-energy ($T \simeq 0$) effective theory governed by the equations of motion (34)–(35) describes a gauge field which evolves in a classical scalar medium, acquiring mass and producing in turn density fluctuations in this medium. This is in fact a natural mechanism for particles to get mass. Indeed experience shows that particles in dense media respond with a larger inertia.

In the broken phase, the effective Lagrangian density governing the dynamics of the quantum field $A_\mu(x)$ in the presence of the scalar background field $\rho(x) = v + \eta(x)$ takes the form

$$\mathcal{L}^q_{\text{eff}} = -\frac{1}{4}\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu} + \frac{1}{2}m_A^2 A_\mu A^\mu + \mathcal{L}_\mathrm{I} \tag{64}$$

where

$$\mathcal{L}_\mathrm{I} = g m_A \eta A_\mu A^\mu + \frac{1}{2}g^2\eta^2 A_\mu A^\mu \tag{65}$$

describes the interaction of $A_\mu(x)$ with the external classical field $\eta(x)$. In turn, the decoupled evolution of the hidden scalar medium becomes governed by the effective Lagrangian density

$$\mathcal{L}^c_{\text{eff}} = \frac{1}{2}(\partial_\mu\eta)(\partial^\mu\eta) - \left(V_{\text{eff}}|_{\langle\rangle} + \frac{1}{2}m^2\eta^2 - \frac{\lambda}{3}\eta^3 + \ldots\right) \tag{66}$$

which, as previously stated, defines a mean field theory containing the effects of gauge interactions.

The decoupling of the scalar sector enables us to define the energy density of the medium

$$\mathcal{H}^c_{\text{eff}} = \frac{1}{2}\dot{\eta}^2 + \frac{1}{2}(\nabla\eta)^2 + V_{\text{eff}}(x) \tag{67}$$

so that, according to Eq.(62), the vacuum energy density turns out to be

$$\langle 0|\mathcal{H}^c_{\text{eff}}|0\rangle \simeq \mu n_0 \tag{68}$$

indicating that, to a good approximation, the chemical potential of the Bose superfluid at $T \simeq 0$ coincides with the mass μ.

From Eqs.(64)–(66) we see that nonperturbative interactions with those particles condensed in the zero-momentum state have been absorbed into the physical parameters characterizing the low-energy effective theory, $\mathcal{L}^q_{\text{eff}} \oplus \mathcal{L}^c_{\text{eff}}$, which now describes elementary excitations (particles) with respect to the nontrivial asymmetric vacuum (ether). As a result, the vacuum degrees of freedom disappear from the formulation. The physical reason for this is related with the nonperturbative decoupling of the scalar sector. Indeed, according to Eqs.(26) and (55), in the broken phase the effective couplings become suppressed by inverse powers of the number density n_0 of particles condensed in the ground state, so that in the presence of a macroscopically large number of quanta, $n_0 \to \infty$, the scalar field decouples and then behaves as a classical field, just in agreement with physical expectations. Under these conditions the condensate appears as a *rigid* (i.e., unperturbed) macroscopic classical body, so that, as usually happens in these cases, its effects only enters the formulation in a static way.

On the other hand, as evident from Eq.(26), in the limit $n_0 \to \infty$ the scalar sector not only decouples but also becomes non-interacting (*trivial*). It defines a nonperturbative low-temperature free scalar theory. However, due to the fact that in the present mechanism the symmetry breakdown occurs as a direct consequence of gauge interactions (rather than as

a consequence of self-interactions in the scalar sector) obviously the *triviality* of the theory poses no problem here.

It should be emphasized that the low-energy effective theory $\mathcal{L}^q_{\rm eff} \oplus \mathcal{L}^c_{\rm eff}$ is only adequate for describing the broken phase characterized by the nonvanishing order parameter $\langle \rho \rangle_0 \equiv v \neq 0$. In fact, both Eq.(12) and the expansion (26) are nonanalytic in the point $v = 0$, indicating the nonperturbative character of our treatment. The physical reason for this behaviour is quite clear. Since the vanishing of the order parameter corresponds to the restoration of gauge symmetry, the limit $v \to 0$ represents a phase transition from the broken phase to the symmetric one. However, experience shows that phase transitions are in general quite complex physical phenomena where the physical properties of the system usually change drastically. Under these conditions it is not possible to continously pass from one to another phase and a specific mathematical formulation is required for describing the asymmetric phase. In fact the appearance of a nonvanishing order parameter determines the emergence of new physics at the low-temperature scale. A stable, highly-ordered macroscopic medium with a clear classical behaviour appears as an *emergent property* (i.e., a property of a complex system which is not contained in its parts [1]).

It should be noticed, however, that a proper treatment of the phase transition leading to the restoration of gauge symmetry would require a finite-temperature formalism. The possibility of phase transitions in gauge theories was first suggested by Kirzhnitz [20] and is treated in detail, within the framework of the Higgs mechanism, in Refs. [21–24].

To sum up, starting from a gauge-invariant Lagrangian density $\mathcal{L}$ we have obtained, as a particular solution of the equations of motion, a low-energy effective theory, $\mathcal{L}^q_{\rm eff} \oplus \mathcal{L}^c_{\rm eff}$, where $\mathcal{L}^q_{\rm eff}$ describes massive gauge bosons in a scalar medium and would be relevant to particle physics, while $\mathcal{L}^c_{\rm eff}$ governs the decoupled dynamics of the medium and would be relevant to cosmology.

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